Topological Design Sensitivity Analysis for Ferrite Magnet Motor Design

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This paper presents a ferrite magnet motor design by considering the topological design sensitivity. A level set based topology optimization technique is introduced to determine the drawbacks of heuristic or parametric optimization methods such as dependence on the initial shape and lack of detailed representation of the boundaries. For multi-material structure design, N level set functions are required to represent 2^N different materials. Therefore, two level-set functions are required to design a ferrite magnet motor consisting of three different materials: a permanent magnet, ferromagnetic material, and air. Six cases are revealed with different sensitivity values depending on the selection of the material domains. To compare the six different cases, an optimization problem is formulated that minimizes the sum of the difference of the target torque. It is confirmed that the selection of material domains, which has a low difference value of sensitivity, is a requirement to achieve optimum design with better performance and better convergence.

***Index Terms*—Ferrite magnet motor, Level set method, Sensitivity analysis, Topology optimization**

I. Introduction

THE permanent magnet synchronous motor (PMSM) with a rare-earth magnet has been widely used in industrial fields due to its high power density. However, the unstable rare-earth magnet supply rate is causing its price to fluctuate; hence, a design technique for replacing the rare-earth magnet with a ferrite magnet in PMSM has recently become a critical issue [1]-[2]. To achieve this, an optimization method has to be conducted in the design stage of the motor to achieve target performance since the residual flux density of the ferrite magnet is about one-third of that of a rare-earth magnet. Therefore, there has been a research effort to determine the optimal shape of the magnet by implementing topology optimization.

The aim of this paper is to present and discuss the design of a ferrite magnet motor using a level set based topology optimization method considering the different expressions of multi-materials such as a ferrite magnet, ferromagnetic material, and air. Level set based topology optimization [3] is employed to determine the innovative material distributions, and this requires N level set functions to represent the 2^N different materials [4]. Therefore two level set functions are required to design PMSM consisting of three materials. It is important to define which material is used in which material domain since the design sensitivities for each case result in different numerical values. Six cases are revealed with different sensitivity values depending on the selection of the material domains. In order to compare the six cases, an optimization problem is formulated to minimize the sum of the difference of the target torque.

II. Problem Formulation

A. Material expression using level set functions

The sign of the level set function (ϕ) is used to classify the multi-material domains as shown in Fig. 1. To represent the material properties for the magneto-static analysis, the relative magnetic reluctivity (v_r) and the residual flux density (B_r) are defined by the level set functions (ϕ) as follows:

$$v_r(\phi_1,\phi_2) = v_1\chi_1\chi_2 + v_2\chi_1(1-\chi_2) + v_3(1-\chi_1)\chi_2 + v_4(1-\chi_1)(1-\chi_2) \quad (1)$$

$$\mathbf{B}_r(\phi_1,\phi_2,\phi_3) = \begin{pmatrix} B_1\chi_1\chi_2 + B_2\chi_1(1-\chi_2) \\ +B_3(1-\chi_1)\chi_2 + B_4(1-\chi_1)(1-\chi_2) \end{pmatrix} \begin{pmatrix} \cos((1+\phi_3)\pi) \\ \sin((1+\phi_3)\pi) \end{pmatrix} \quad (2)$$

where the characteristic function (χ) is obtained by:

$$\chi_i = \begin{cases} 0 \text{ for } \phi_i < 0 \\ 1 \text{ for } \phi_i \geq 0 \end{cases} \quad \text{where } -1 \leq \phi_i \leq 1 \quad i = 1,2,3 \quad (3)$$

B. Sensitivity analysis

The topological derivative of the objective function (F) in the level-set based topology optimization method [5] is defined by

$$d_t F = -\frac{F(\chi_i)}{d\chi_i} \quad (4)$$

By applying the adjoint variable method for the magneto-static problem, the design sensitivity can be derived as follows:

$$\begin{aligned} \tilde{F}(\chi_i) &= F(\chi_i) - \lambda^T R(\chi_i) \\ \frac{d\tilde{F}(\chi_i)}{d\chi_i} &= \frac{\partial F(\chi_i)}{\partial \mathbf{A}_z}\frac{d\mathbf{A}_z}{d\chi_i} - \lambda^T\left(\frac{\partial R(\chi_i)}{\partial \chi_i} + \frac{\partial R(\chi_i)}{\partial \mathbf{A}_z}\frac{d\mathbf{A}_z}{d\chi_i}\right) \\ &= \left(\frac{\partial F(\chi_i)}{\partial \mathbf{A}_z} - \lambda^T\frac{\partial R(\chi_i)}{\partial \mathbf{A}_z}\right)\frac{d\mathbf{A}_z}{d\chi_i} - \lambda^T\frac{\partial R(\chi_i)}{\partial \chi_i} \\ &= \frac{\lambda^T}{\mu_0}\left[\frac{\partial v_r(\chi_i)}{\partial \chi_i}\left(\nabla^2\mathbf{A}_z + \nabla\times\mathbf{B}_r(\chi_i)\right) + v_r(\chi_i)\nabla\times\frac{\partial \mathbf{B}_r(\chi_i)}{\partial \chi_i}\right] \end{aligned} \quad (5)$$

where

$$\lambda^T = \frac{F(\chi_i)}{\partial \mathbf{A}_z}\left[\frac{\partial R(\chi_i)}{\partial \mathbf{A}_z}\right]^{-1}$$

$$R(\chi_i) = -\frac{v_r(\chi_i)}{\mu_0}(\nabla^2\mathbf{A}_z + \nabla\times\mathbf{B}_r(\chi_i)) - J_z = 0$$

where $\mathbf{A_z}$ is the magnetic vector potential, R is the governing equation of the magneto-static field including permanent magnet, and μ_0 is the magnetic permeability of air.

The term $\frac{\partial v_r(\chi_i)}{\partial \chi_i}$ in the design sensitivity has different values depending on the selection of material domains as follows, and these results make a difference in the optimum design.

$$\frac{\partial v_r(\chi_i)}{\partial \chi_i} = \begin{cases} (v_1 - v_3)\chi_2 + (v_2 - v_4)(1-\chi_2) & for\ \phi_1 \\ (v_1 - v_2)\chi_1 + (v_3 - v_4)(1-\chi_1) & for\ \phi_2 \end{cases} \tag{6}$$

The detailed expression of (6) is summarized in Table 1.

III. Design Example and optimization results

The proposed method is applied to the entire rotor domain of 8 pole-12 slot interior permanent magnet motor for operating electric power steering. For the magnetic analysis and optimization process, the material properties of $v_A = 1$, $v_P = 1/1.05$, $v_F = 1/100\sim1/6000$ and $B_r = 0.4$ T and a constant of $T_{\text{target}} = 2.1$ Nm are used. An optimization problem that minimizes the sum of the differences of the target torque is formulated.

$$\min_{\phi_1,\phi_2,\phi_3} F(\phi_1,\phi_2,\phi_3) = \sum_{i=0}^{n} \left(\frac{T_i(\phi_1,\phi_2,\phi_3)}{T_{\text{target}}} - 1\right)^2 \tag{7}$$

where n is the rotating angle of the rotor and T_{target} is the target torque.

Topology optimization results and objective function histories are illustrated in Fig. 2 and Fig. 3, respectively. Case 1, which has the same value of (6) among the level set functions and the lowest difference value of (6) in each level set function, has the lowest objective function value and fluctuates in design iteration. In contrast, case 3, which has a large difference value of (6) has large fluctuates in design iteration. Also, Case 4, which has the same sensitivity value of (6) among the level set functions, has lower objective function value and fluctuation than other cases 2, 3, 5 and 6.

Based on the optimum results, it is confirmed that the selection of material domain, which has a low difference value of (6) among the level-set functions, such as case 1 and 4, is requirement to achieve the optimum design has with better performance with and better convergence.

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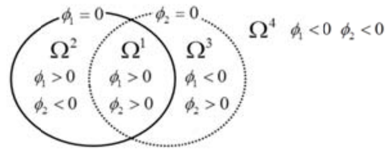

Fig. 1. Different material domains of two level set functions.

TABLE I
SENSITIVITY OF THE RELATIVE MAGNETIC RELUCTIVITY

ID	Material domain Ω^1	Ω^2	Ω^3	Ω^4	$\partial v_r / \partial \chi_1$ $(\phi_2 > 0)$	$\partial v_r / \partial \chi_1$ $(\phi_2 < 0)$	$\partial v_r / \partial \chi_2$ $(\phi_1 > 0)$	$\partial v_r / \partial \chi_2$ $(\phi_1 < 0)$
1	P	F	F	A	$v_P - v_F$	$v_F - v_A$	$v_P - v_F$	$v_F - v_A$
2	P	F	A	A	$v_P - v_A$	$v_F - v_A$	$v_P - v_F$	0
3	P	F	A	P	$v_P - v_A$	$v_F - v_P$	$v_P - v_F$	$v_A - v_P$
4	P	A	A	F	$v_P - v_A$	$v_A - v_F$	$v_P - v_A$	$v_A - v_F$
5	P	F	A	F	$v_P - v_A$	0	$v_P - v_F$	$v_A - v_F$
6	P	P	F	A	$v_P - v_F$	$v_P - v_A$	0	$v_F - v_A$

P: permanent magnet, F: ferromagnetic material and A: air

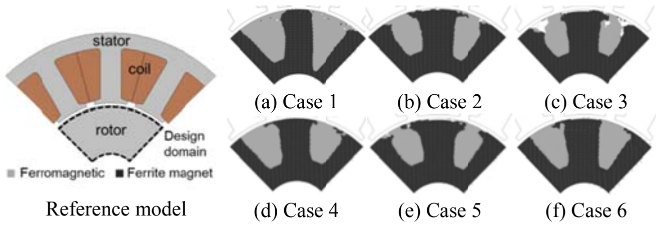

Fig. 2. Topology optimization results

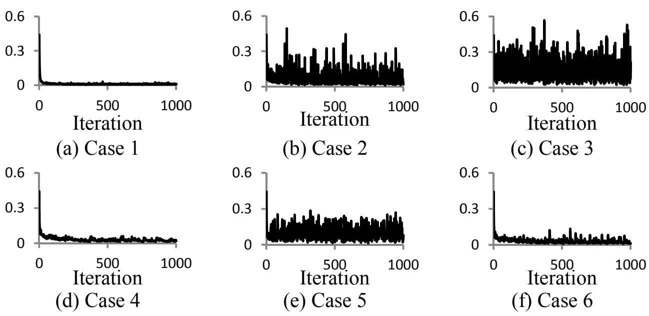

Fig. 3. Objective function history

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